Jet broadening in deeply inelastic scattering

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(Received 10 April 1998; published 9 November 1998)

In deeply inelastic lepton-nucleus scattering (DIS), the average jet transverse momentum is broadened because of the multiple scattering in the nuclear medium. The size of jet broadening is proportional to the multiparton correlation functions inside nuclei. We show that, at leading order, jet broadening in DIS and nuclear enhancement in a dijet momentum imbalance and averaged Drell-Yan transverse momentum square, $\langle q_T^2\rangle$, share the same four-parton correlation functions. We argue that jet broadening in DIS provides an independent measurement of the four-parton correlation functions and a test of the QCD treatment of multiple scattering. [S0556-2821(98)08021-7]

PACS number(s): 13.87.Ce, 12.38.Bx, 13.60.Hb, 13.85.Qk

I. INTRODUCTION

When a parton propagates through nuclear matter, the average transverse momentum may be broadened because of the multiple scattering of the parton. The nuclear dependence of such broadening provides an excellent probe to study QCD dynamics beyond the single hard-scattering picture. A reliable calculation of multiple scattering in QCD perturbation theory requires to extend the factorization theorem [1] beyond the leading power. Qiu and Sterman showed that the factorization theorem for hadron-hadron scattering holds at the first-nonleading power in momentum transfer [2], which is enough to study double scattering processes in hadronic collisions. According to this generalized factorization theorem, the double scattering contribution can be expressed in terms of universal four-parton (twist-four) correlation functions. The predictions of the multiple scattering effects rely on accurate information of the four-parton correlation functions. Various estimates of the size of these four-parton correlation functions were derived recently [3–5]. It is the purpose of this paper to show that jet broadening in deeply inelastic lepton-nucleus scattering (DIS) can be used as an independent process to test the previous estimate of the four-parton correlation functions.

The four-parton (twist-4) correlation functions are as fundamental as the normal twist-2 parton distributions. While twist-2 parton distributions have the interpretation of the probability distributions to find a parton within a hadron, four-parton correlation functions provide information on quantum correlations of multipartons inside a hadron. Just like normal twist-2 parton distributions, multiparton correlation functions are nonperturbative. QCD perturbation theory cannot provide the absolute prediction of these correlation functions. But, due to factorization theorems, these correlation functions are universal; and can be measured in some processes and be tested in other processes.

Luo, Qiu and Sterman (LQS) calculated the nuclear dependence of dijet momentum imbalance in photon-nucleus collision in terms of nuclear four-parton correlation functions [3]:

$$T_{q/A}(x)=\int \frac{dy^-}{2\pi}e^{ixp^+y^-}\frac{dy_1^-dy_2^-}{2\pi}\times\theta(y_1^- - y^-)\,\theta(y_2^-)\frac{1}{2}\langle p_A|\overline{\psi}_q(0)\,\gamma^+F_\sigma{}^+(y_2^-)\times F^{+\sigma}(y_1^-)\,\psi_q(y^-)|p_A\rangle; \tag{1a}$$

and

$$T_{g/A}(x)=\int \frac{dy^-}{2\pi}e^{ixp^+y^-}\frac{dy_1^-dy_2^-}{2\pi}\times\theta(y_1^- - y^-)\,\theta(y_2^-)\frac{1}{xp^+}\langle p_A|F_\alpha{}^+(0)F_\sigma{}^+(y_2^-)\times F^{+\sigma}(y_1^-)F^{+\alpha}(y^-)|p_A\rangle. \tag{1b}$$

By comparing the operator definitions of these four-parton correlation functions and the definitions of the normal twist-2 parton distributions, LQS proposed the following model [3,6]:

$$T_{f/A}(x)=\lambda^2A^{4/3}\phi_{f/N}(x), \tag{2}$$

where $\phi_{f/N}(x)$ with $f=q,\overline{q},g$ are the normal twist-2 parton distribution of a nucleon, and λ is a free parameter to be fixed by experimental data. Using the Fermilab E683 data on dijet momentum imbalance, LQS estimated the size of the four-parton correlation functions to be of the order $\lambda^2\approx 0.05$–0.1 GeV^2 [3]. The higher value $\lambda^2\approx 0.1$ GeV^2 was obtained by keeping only the valence quark contribution, while the lower value $\lambda^2\approx 0.05$ GeV^2 was obtained by keeping only the gluonic contribution. Since both the quark and the gluon initiated subprocesses contributing to the momentum imbalance, dijet data indicate that λ^2 should be between 0.05 GeV^2 and 0.1 GeV^2 [3].

The nuclear enhancement of the average Drell-Yan transverse momentum, $\Delta\langle q_T^2\rangle$, also depends on the similar four-parton correlation functions [4]. However, Fermilab and CERN data on nuclear enhancement of the average Drell-Yan transverse momentum, $\Delta\langle q_T^2\rangle$, prefer a much smaller size of the four-parton correlation functions [4,7,8]. Actually, as we will show below, the Drell-Yan data favor the four-parton correlation functions about five times smaller than what was extracted from the dijet data. This discrepancy may result from different higher order contribution to nuclear enhancement of the average Drell-Yan transverse momentum and the dijet momentum imbalance. We have pure initial-state multiple scattering for the Drell-Yan process, while for a dijet system, we have pure final state multiple scattering. It is necessary to study the higher order corrections to these two observables in order to test QCD treatment of multiple scattering. It was argued in Refs. [9,10] that the higher order contribution to the Drell-Yan nuclear enhancement is important. Meanwhile, it is also important and necessary to find different observables that depend on the same four-parton correlation functions.

We show in this paper that jet broadening in deeply inelastic lepton-nucleus scattering provides an independent measurement of the four-parton correlation functions. At the leading order in deeply inelastic scattering, a quark scattered of the virtual photon forms a jet (known as the current jet) in the final state. Because of the multiple interaction when the scattered parton propagates through the nuclear matter, the transverse momentum of the final state jet is broadened in deeply inelastic lepton-nucleus scattering [11]. We show below that the size of the jet broadening is directly proportional to the four-parton correlation functions. These four-parton correlation functions are the same as those appeared in the nuclear enhancement of the dijet momentum imbalance. Current data for the dijet momentum imbalance and nuclear enhancement of average Drell-Yan transverse momentum, $\Delta\langle q_T^2\rangle$, provide two different sizes of the four-parton correlation functions. Using these data, without additional parameters, we predict a range of the jet broadening in DIS. Future experiments at the DESY ep collider HERA with nuclear beams can provide a direct test of QCD treatment of the multiple scattering.

The rest of this paper is organized as follows. In Sec. II, we derive the nuclear enhancement of the average Drell-Yan transverse momentum by using the same method used by LQS to derive the dijet momentum imbalance, and show that our result is consistent with that presented in Ref. [4]. In Sec. III, using the same method, we derive jet broadening in DIS in terms of the same four-parton correlation functions. Finally, in Sec. IV, we use the values of λ^2 extracted from data on dijet momentum imbalance and nuclear enhancement of average Drell-Yan transverse momentum to estimate jet broadening in DIS. We also present our discussions and conclusions in this section.

II. NUCLEAR ENHANCEMENT OF AVERAGE DRELL-YAN TRANSVERSE MOMENTUM

Consider the Drell-Yan process in hadron-nucleus collisions, $h(p')+A(p)\to l^+l^-(q)+X$, where q is the four-momentum for the virtual photon γ^* which decays into the lepton pair. p' is the momentum for the incoming beam hadron and p is the momentum per nucleon for the nucleus with the atomic number A.

Let q_T be the transverse momentum of the Drell-Yan pair, we define the averaged transverse momentum square as

$$\langle q_T^2\rangle^{hA}=\int dq_T^2 q_T^2\frac{d\sigma_{hA}}{dQ^2dq_T^2}\Big/\frac{d\sigma_{hA}}{dQ^2}. \tag{3}$$

In Eq. (3), Q is the total invariant mass of the lepton pair with $Q^2=q^2$. The transverse momentum spectrum, $d\sigma/dQ^2dq_T^2$, is sensitive to the multiple scattering, and has the $A^{1/3}$-type nuclear size effect. We can write the cross section as

$$\sigma_{hA}=\sigma_{hA}^S+\sigma_{hA}^D+\cdots, \tag{4}$$

where σ_{hA}^S, σ_{hA}^D, and "$\cdots$" represent the single, double and higher multiple scattering, respectively. The single scattering is localized, hence, σ_{hA}^S does not have a large dependence on the nuclear size. Therefore, $\sigma_{hA}^S\approx A\sigma_{hN}$ with N represents a nucleon. As we will demonstrate explicitly, the inclusive cross section $d\sigma/dQ^2$ has a much weaker nuclear dependence after integration over the whole momentum spectrum. Hence,

$$\int dq_T^2 q_T^2\frac{d\sigma_{hA}^S}{dQ^2dq_T^2}\Big/\frac{d\sigma_{hA}}{dQ^2}\approx\langle q_T^2\rangle^{hN}. \tag{5}$$

In order to extract the effect due to the multiple scattering, we introduce nuclear enhancement of the average Drell-Yan transverse momentum $\langle q_T^2\rangle$ as

$$\Delta\langle q_T^2\rangle\equiv\langle q_T^2\rangle^{hA}-\langle q_T^2\rangle^{hN}. \tag{6}$$

If we keep only the double scattering contribution and neglect contribution from the higher multiple scattering, we have

$$\Delta\langle q_T^2\rangle\approx\int dq_T^2 q_T^2\frac{d\sigma_{hA}^D}{dQ^2dq_T^2}\Big/\frac{d\sigma_{hA}}{dQ^2}. \tag{7}$$

From our definition, $\Delta\langle q_T^2\rangle$ represents a measurement of QCD dynamics beyond the traditional single-hard scattering picture. The nuclear enhancement of average Drell-Yan transverse momentum, $\Delta\langle q_T^2\rangle$ defined in Eq. (6), is a result of the multiple scattering between the parton from incoming beam and the nuclear matter before the Drell-Yan pair is produced.

Since we are interested in the averaged transverse momentum square, we do not need to know the angular information of the Drell-Yan lepton pair. After integration over the lepton's angular information, the Drell-Yan cross section can be written as

$$d\sigma_{hA\to l^+l^-}=\left(\frac{2\alpha_{em}}{3Q^2}\right)(-g_{\mu\nu}W^{\mu\nu}_{hA\to\gamma^*}(q)), \tag{8}$$

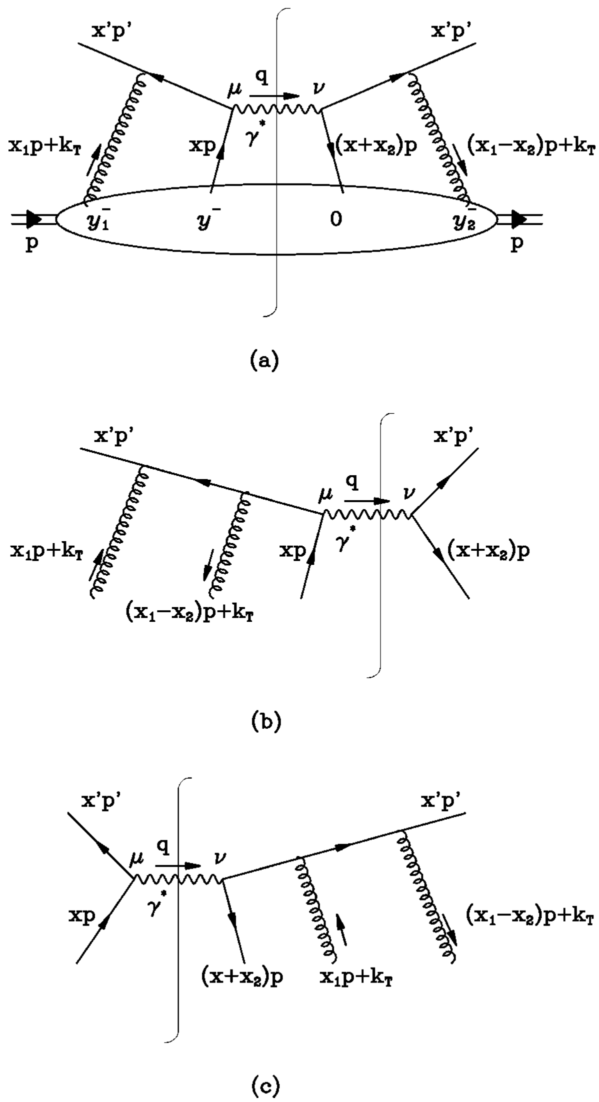

FIG. 1. Lowest order double scattering contribution to the nuclear enhancement of Drell-Yan $\langle q_T^2\rangle$; (a) symmetric diagram; (b) and (c): interference diagrams.

where $W^{\mu\nu}$ is the hadronic tensor [12]. To simplify the notation of following derivation, we introduce

$$W_{hA\to\gamma^*}(q)\equiv -g_{\mu\nu}W^{\mu\nu}_{hA\to\gamma^*}(q). \tag{9}$$

At the leading order, the single scattering contribution to $W_{hA\to\gamma^*}(q)$ is given by

$$W^S_{hA\to\gamma^*}(q)=\sum_q \int dx'\, \phi_{\bar{q}/h}(x') \times\int dx\, \phi_{q/A}(x)\left[e_q^2\frac{2\pi\alpha_{em}}{3}\right], \tag{10}$$

where superscript ''S'' signals the single scattering; and the corresponding inclusive single scattering cross section is [12]

$$\frac{d\sigma_{hA\to l^+l^-}}{dQ^2}=\sigma_0\sum_q e_q^2\int dx'\, \phi_{\bar{q}/h}(x') \times\int dx\, \phi_{q/A}(x)\,\delta(Q^2-xx's), \tag{11}$$

with $s=(p+p')^2$ and the Born cross section

$$\sigma_0=\frac{4\pi\alpha_{em}^2}{9Q^2}. \tag{12}$$

Consider only double scattering inside the nuclear target, we can factorize the double scattering contribution to $W(q)$ as

$$W^D_{hA\to\gamma^*}(q)=\sum_f \int dx'\, \phi_{f/h}(x')\hat{W}^D_{fA\to\gamma^*}(x',q), \tag{13}$$

where superscript ''D'' indicates the double scattering contribution, and f sums over all parton flavors. In Eq. (13), $\hat{W}^D_{fA\to\gamma^*}(x',D)$ is the hadronic contribution from the double scattering between a parton f and the nucleus. At the leading order in α_s, $\hat{W}^D_{fA\to\gamma^*}(x',D)$ is given by the Feynman diagrams in Fig. 1. According to the generalized factorization theorem [2], $\hat{W}^D_{fA\to\gamma^*}(x',D)$ can be factorized as

$$\hat{W}^D_{bA\to\gamma^*}(x',D)=\frac{1}{2x's}\int dx dx_1 dx_2\int dk_T^2\overline{T}^{(I)}(x,x_1,x_2,k_T) \times\overline{H}(x,x_1,x_2,k_T,x'p',p,q)\Gamma^D, \tag{14}$$

where Γ^D is the phase space factor for the double scattering. For the diagram shown in Fig. 1(a), for example, the phase space factor is given by

$$\Gamma^D_a=\frac{d^4q}{(2\pi)^4}(2\pi)^4\delta^4(xp+x_1p+x'p'+k_T-q) =\frac{1}{x's}\delta\left(x+x_1-\frac{k_T^2}{x's}-\frac{Q^2}{x's}\right)\delta(q_T^2-k_T^2)dQ^2dq_T^2. \tag{15}$$

At this stage of the derivation, $\overline{T}$ and $\overline{H}$ in Eq. (14) both depend on gauge choice, even though $W(q)^D$ is gauge invariant. In this paper, Feynman gauge is used in our calculation.

In Eq. (14), the hadronic matrix element

$$\overline{T}^{(I)}(x,x_1,x_2,k_T) = \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi}\frac{d^2y_T}{(2\pi)^2} e^{ixp^+y^-} e^{ix_1p^+(y_1^- - y_2^-)} \times e^{ix_2p^+y_2^-} e^{ik_T\cdot y_T}\frac{1}{2}\langle p_A|A^+(y_2^-,0_T)\overline{\psi}_q(0)\gamma^+ \times \psi_q(y^-)A^+(y_1^-,y_{1T})|p_A\rangle. \quad (16)$$

Because of the exponential factors in Eq. (16), the position space integration, dy^-'s cannot give large dependence on the nuclear size unless the parton momentum fraction in one of the exponentials vanishes. If the exponential vanishes, the corresponding position space integration can be extended to the size of the whole nucleus. Therefore, in order to get large nuclear enhancement or jet broadening, we need to consider only Feynman diagrams that can provide poles which set parton momentum fractions on the exponentials to be zero [3]. At the leading order, only diagrams shown in Fig. 1 have the necessary poles. These diagrams contribute to the double scattering partonic part $\overline{H}(x,x_1,x_2,k_T,x'p',p,q)$ in Eq. (14).

For the leading order diagrams shown in Fig. 1, the corresponding partonic parts have two possible poles. For example, for the diagram shown in Fig. 1, the partonic part has the following general structure:

$$\overline{H}^a \sim \frac{1}{(x'p'+x_1p+k_T)^2+i\epsilon} \times \frac{1}{(x'p'+(x_1-x_2)p+k_T)^2-i\epsilon} \times(-g_{\mu\nu})\hat{H}^{\mu\nu} \propto \frac{1}{x_1-\frac{k_T^2}{x's}+i\epsilon} \times \frac{1}{x_1-x_2-\frac{k_T^2}{x's}-i\epsilon}, \quad (17)$$

where $\hat{H}^{\mu\nu}$ is the numerator which is proportional to a quark spinor trace.

The first step to evaluate Eq. (14) is to carry out the integration over parton momentum fractions $dxdx_1dx_2$. Using one of the δ-functions in phase space factor in Eq. (15) to fix dx integration, and the two poles in the partonic part in Eq. (17) to perform the contour integration for dx_1dx_2, we obtain

$$I_a \equiv \int dxdx_1dx_2 e^{ixp^+y^-} e^{ix_1p^+(y_1^- - y_2^-)} e^{ix_2p^+y_2^-} \frac{1}{x_1-\frac{k_T^2}{x's}+i\epsilon} \times \frac{1}{x_1-x_2-\frac{k_T^2}{x's}-i\epsilon}\delta\left(x+x_1-\frac{k_T^2}{x's}-\frac{Q^2}{x's}\right) = (4\pi^2)\,\theta(y^- - y_1^-)\,\theta(-y_2^-)\,e^{i(\tau/x')p^+y^-} \times e^{i(k_T^2/(x's))p^+(y_1^- - y_2^-)}, \quad (18)$$

with $\tau=Q^2/s$. For the diagram shown in Fig. 1(b), we have a slightly different phase space factor

$$\Gamma_b^D = \frac{1}{x's}\delta\left(x+x_2-\frac{Q^2}{x's}\right)\delta(q_T^2)dQ^2dq_T^2. \quad (19)$$

The corresponding partonic part $\overline{H}^b$ has slightly different poles

$$\overline{H}^b \propto \frac{1}{x_1-\frac{k_T^2}{x's}+i\epsilon}\frac{1}{x_2+i\epsilon}. \quad (20)$$

Integrating over parton momentum fractions, $dxdx_1dx_2$, we have

$$I_b \equiv \int dxdx_1dx_2 e^{ixp^+y^-} e^{ix_1p^+(y_1^- - y_2^-)} e^{ix_2p^+y_2^-} \times \frac{1}{x_1-\frac{k_T^2}{x's}+i\epsilon}\frac{1}{x_2+i\epsilon}\delta\left(x+x_2-\frac{Q^2}{x's}\right) = (-4\pi^2)\,\theta(y_2^- - y_1^-)\,\theta(y^- - y_2^-) \times e^{i(\tau/x')p^+y^-} e^{i(k_T^2/(x's))p^+(y_1^- - y_2^-)}. \quad (21)$$

Similarly, for the diagram shown in Fig 1(c), we have

$$I_c = (-4\pi^2)\,\theta(y_1^- - y_2^-)\,\theta(-y_1^-) \times e^{i(\tau/x')p^+y^-} e^{i(k_T^2/(x's))p^+(y_1^- - y_2^-)}. \quad (22)$$

It is easy to show that the numerator spinor trace gives the same factor for all three diagrams. Therefore, the total contribution to jet broadening is proportional to $I_a+I_b+I_c$, and this sum has the following feature:

$$\frac{d\sigma_{hA}^D}{dQ^2dq_T^2} \propto I_a+I_b+I_c \propto \theta(y^- - y_1^-)\,\theta(-y_2^-) \times[\delta(q_T^2-k_T^2)-\delta(q_T^2)] \quad (23)$$

$$+[\theta(y^- - y_1^-)\theta(-y_2^-) - \theta(y_2^- - y_1^-)\theta(y^- - y_2^-) - \theta(y_1^- - y_2^-)\theta(-y_1^-)]\delta(q_T^2). \tag{24}$$

It is clear from Eq. (24) that for the inclusive Drell-Yan cross section, $d\sigma/dQ^2$, the double scattering contribution, $d\sigma_{hA}^D/dQ^2$, does not have a large dependence on the nuclear size. The integration over dq_T^2 eliminates the first term in Eq. (24), while the second term is localized in space if τ/x' is not too small. When the τ/x' is finite, and p^+ is large, $\exp[i(\tau/x')p^+y^-]$ effectively restricts $y^- \sim 1/((\tau/x')p^+) \rightarrow 0$. When $y^- \rightarrow 0$, the combination of the three pairs of θ-functions in Eq. (24) vanishes,

$$\frac{d\sigma_{hA}^D}{dQ^2} \equiv \int dq_T^2 \left(\frac{d\sigma_{hA}^D}{dQ^2 dq_T^2}\right) \propto [\theta(y^- - y_1^-)\theta(-y_2^-) - \theta(y_2^- - y_1^-)\theta(y^- - y_2^-) - \theta(y_1^- - y_2^-)\theta(-y_1^-)] \rightarrow 0 \quad \text{as } y^- \rightarrow 0. \tag{25}$$

Physically, Eq. (25) says that all integrations of y^-'s are localized. Actually, at the leading order, the term proportional to $\theta(y^- - y_1^-)\theta(-y_2^-) - \theta(y_2^- - y_1^-)\theta(y^- - y_2^-) - \theta(y_1^- - y_2^-)\theta(-y_1^-)$ is the Eikonal contribution in Feynman gauge to make the normal twist-2 quark distribution $\phi_{q/A}(x)$ in Eq. (11) gauge invariant. Equation (25) is a good example to demonstrate that the double scattering does not give a large nuclear size effect to the total inclusive cross section.

On the other hand, from Eq. (24), the double scattering contribution to the averaged transverse momentum square can give a large nuclear size effect

$$\Delta\langle q_T^2\rangle \sim \int dq_T^2\, q_T^2 \left(\frac{d\sigma_{hA}^D}{dQ^2 dq_T^2}\right) \propto \int dq_T^2\, q_T^2 [\delta(q_T^2 - k_T^2) - \delta(q_T^2)] \sim k_T^2. \tag{26}$$

Actually, k_T^2 in Eq. (26) needs to be integrated first. But Eq. (26) already demonstrates that $\Delta\langle q_T^2\rangle$ is proportional k_T^2, which is the kick of the transverse momentum the parton received from the additional scattering. The bigger the nuclear size, the bigger the effective k_T^2. As shown below, $\Delta\langle q_T^2\rangle$ is proportional to the nuclear size.

After demonstrating the physical picture of transverse momentum broadening in the Drell-Yan process, we are now ready to carry out the algebra. Working out the spinor trace for all three diagrams in Fig. 1, and combining three diagrams and dropping the term localized in space, we obtain

$$\Delta\langle q_T^2\rangle \frac{d\sigma_{hA}}{dQ^2} = \int dq_T^2\, q_T^2 \sigma_0 \sum_q \int dx'\, \phi_{\bar{q}/h}(x') \int dx\, \delta(Q^2 - xx's) \int \frac{dy}{2\pi} \frac{dy_1 dy_2}{2\pi} \theta(y^- - y_1^-)\theta(-y_2^-) e^{ixp^+y^-} \frac{d^2 y_T}{(2\pi)^2}$$
$$\times \int d^2 k_T e^{ik_T\cdot y_T} e^{i(k_T^2/(x's))p^+(y_1^- - y_2^-)} \frac{1}{2} \langle p_A | A^+(y_2^-, 0_T) \bar{\psi}_q(0) \gamma^+ \psi_q(y^-) A^+(y_1^-, y_{1T}) | p_A\rangle$$
$$\times \left(\frac{4\pi^2\alpha_s}{3}\right) [\delta(q_T^2 - k_T^2) - \delta(q_T^2)] \tag{27}$$

$$\approx \sigma_0 \left(\frac{4\pi^2\alpha_s}{3}\right) \sum_q \int dx'\, \phi_{\bar{q}/h}(x') \int dx\, T_{q/A}^{(I)}(x)\, \delta(Q^2 - xx's), \tag{28}$$

where σ_0 is defined in Eq. (12) and the four-parton correlation function is given by

$$T_{q/A}^{(I)}(x) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \int \frac{dy_1^- dy_2^-}{2\pi} \theta(y^- - y_1^-)\theta(-y_2^-) \times \frac{1}{2} \langle p_A | F_\alpha{}^+(y_2^-) \bar{\psi}_q(0) \gamma^+ \psi_q(y^-) F^{+\alpha}(y_1^-) | p_A\rangle. \tag{29}$$

In deriving Eq. (28), we expend $\delta(q_T^2 - k_T^2)$ at $k_T = 0$, known as the collinear expansion, and keep only the first nonvanishing term which corresponds to the second order derivative term,

$$\delta(q_T^2 - k_T^2) - \delta(q_T^2) \approx -\delta'(q_T^2)(-g_{\alpha\beta}^\perp) k_T^\alpha k_T^\beta. \tag{30}$$

We use the factor $k_T^\alpha k_T^\beta$ in Eq. (30) to convert the $k_T^\alpha A^+ k_T^\beta A^+$ into field strength $F^{\alpha+}F^{\beta+}$ by partial integration. Here, we work in Feynman gauge. The terms associated with other components of A^ρ are suppressed by $1/p^+$ compared to those with A^+, because of the requirement of the Lorentz boost invariance for the matrix elements [6]. $T_{q/A}^{(I)}(x)$ defined in Eq. (29) is the four-parton correlation function in a nucleus. The

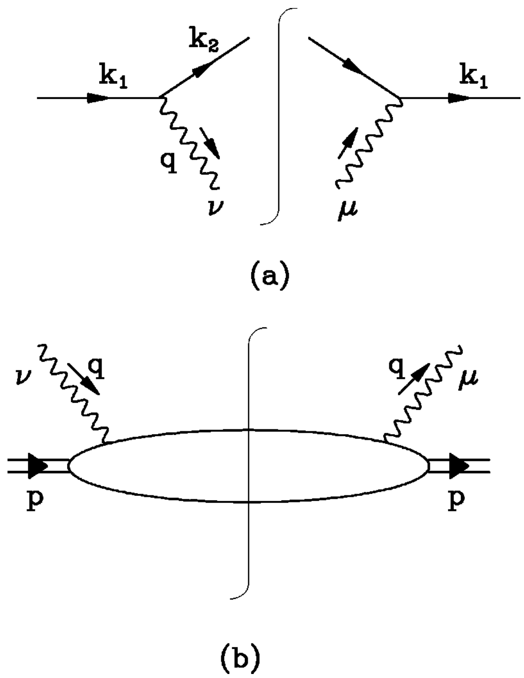

FIG. 2. Diagrams for DIS: (a) diagram representing $L_{\mu\nu}$; (b) diagram representing $W^{\mu\nu}$.

superscript (I) represents the initial state interaction, in order to distinguish from the similar four-parton correlation function defined in Eq. (1a). More discussion on the relation between $T_{q/A}(x)$ and $T^{(I)}_{q/A}(x)$ will be given in Sec. IV.

Combining Eqs. (6), (11), and (28), we obtain the nuclear enhancement of average Drell-Yan transverse momentum at leading order in α_s:

$$\Delta\langle q_T^2\rangle=\left(\frac{4\pi^2\alpha_s}{3}\right)\frac{\sum_q e_q^2\int dx'\,\phi_{\bar{q}/h}(x')T^{(I)}_{q/A}(\tau/x')/x'}{\sum_q e_q^2\int dx'\,\phi_{\bar{q}/h}(x')\phi_{q/A}(\tau/x)/x'}. \tag{31}$$

III. JET BROADENING IN DEEPLY INELASTIC SCATTERING

Consider the jet production in the deeply inelastic lepton-nucleus scattering, $e(k_1)+A(p)\rightarrow e(k_2)+\text{jet}(l)+X$. k_1 and k_2 are the four momenta of the incoming and the outgoing leptons, respectively, and p is the momentum per nucleon for the nucleus with the atomic number A. With l being the observed jet momentum, we define the averaged jet transverse momentum square as

$$\langle l_T^2\rangle^{eA}=\int dl_T^2 l_T^2\frac{d\sigma_{eA}}{dx_B dQ^2 dl_T^2}\Bigg/\frac{d\sigma_{eA}}{dx_B dQ^2}, \tag{32}$$

where $x_B=Q^2/(2p\cdot q)$, $q=k_1-k_2$ is the momentum of the virtual photon, and $Q^2=-q^2$. The jet transverse momentum l_T depends on our choice of the frame. We choose the Breit frame in the following calculation. Similar to the Drell-Yan transverse momentum spectrum, $d\sigma/dQ^2dq_T^2$, the jet transverse momentum spectrum, $d\sigma/dx_BdQ^2dl_T^2$, is sensitive to the $A^{1/3}$ type nuclear size effect due to the multiple scattering. On the other hand, the inclusive DIS cross section $d\sigma/dx_BdQ^2=\int dl_T^2\,d\sigma/dx_BdQ^2dl_T^2$ does not have the $A^{1/3}$ power enhancement. Instead, it has a much weaker A dependence, such as the EMC effect and the nuclear shadowing. To separate the multiple scattering contribution from the single scattering, we define the jet broadening as

$$\Delta\langle l_T^2\rangle\equiv\langle l_T^2\rangle^{eA}-\langle l_T^2\rangle^{eN}. \tag{33}$$

Keeping only the contribution from the double scattering, similar to Eq. (6), we have

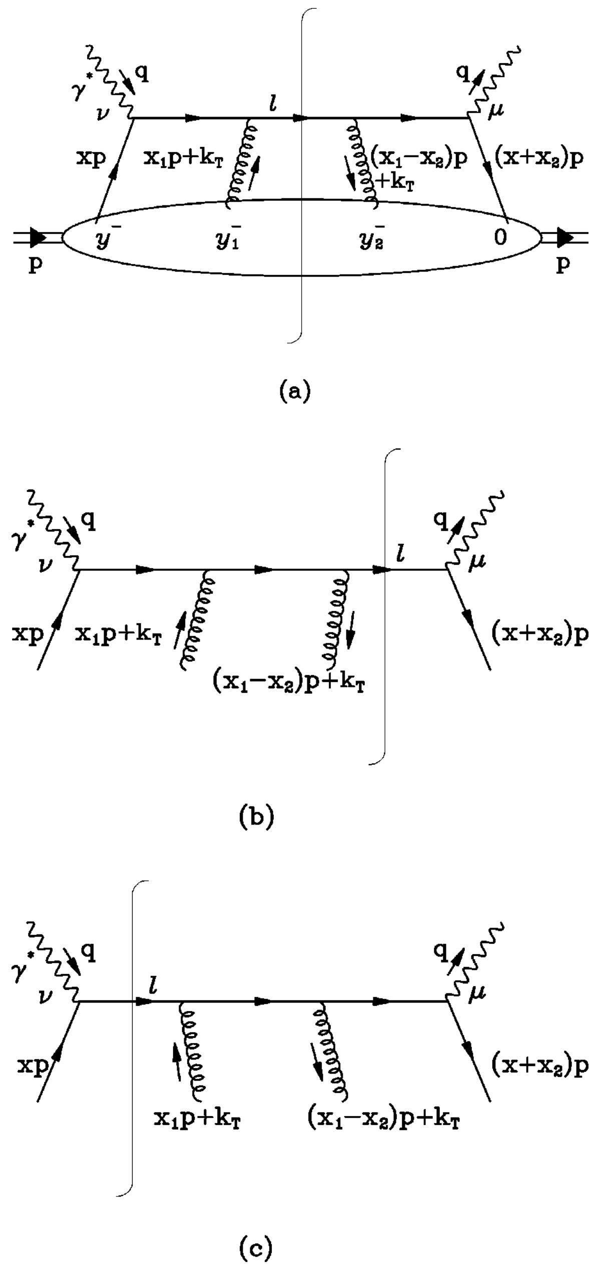

FIG. 3. Lowest order double scattering contribution to jet broadening: (a) symmetric diagram; (b) and (c): interference diagrams.

$$\Delta\langle l_T^2\rangle \approx \int dl_T^2 l_T^2 \frac{d\sigma_{eA}^D}{dx_B dQ^2 dl_T^2} \bigg/ \frac{d\sigma_{eA}}{dx_B dQ^2}. \tag{34}$$

In the rest of this section, we derive the leading contribution to $\Delta\langle l_T^2\rangle$ by using the same technique used to derive nuclear enhancement of the average Drell-Yan transverse momentum, $\Delta\langle q_T^2\rangle$, in last section.

The general expression for the cross section in DIS is

$$d\sigma = \frac{1}{2s}\frac{e^2}{Q^4} L_{\mu\nu} W^{\mu\nu} \frac{d^3k_2}{(2\pi)^3 2E_2}, \tag{35}$$

with $s=(p+k_1)^2$. In Eq. (35), the leptonic tensor $L_{\mu\nu}$ is given by the diagram in Fig. 2(a),

$$L_{\mu\nu} = \frac{1}{2}\mathrm{Tr}(\gamma\cdot k_1 \gamma_\mu \gamma\cdot k_2 \gamma_\nu), \tag{36}$$

and $W^{\mu\nu}$ is the hadronic tensor given by the diagram shown in Fig. 2(b). The leading order double scattering diagrams contributing to jet broadening are given in Fig. 3.

Feynman diagrams in Fig. 3 give the double scattering contribution to the hadronic tensor $W_{\mu\nu}$ as

$$W_{\mu\nu}^D = \sum_a \int dx dx_1 dx_2 \int dk_T^2 \overline{T}^{(F)}(x,x_1,x_2,k_T) \times \overline{H}_{\mu\nu}(x,x_1,x_2,k_T,p,q,l), \tag{37}$$

with the matrix element

$$\overline{T}^{(F)}(x,x_1,x_2,k_T) = \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi}\frac{d^2y_T}{(2\pi)^2} e^{ixp^+y^-} e^{ix_1p^+(y_1^- - y_2^-)} \times e^{ix_2p^+y_2^-} e^{ik_T\cdot y_T} \frac{1}{2}\langle p_A|\overline{\psi}_q(0)\gamma^+ \times A^+(y_2^-,0_T) A^+(y_1^-,y_{1T})\psi_q(y^-)|p_A\rangle, \tag{38}$$

where superscript (F) indicates the final-state double scattering. The matrix element $\overline{T}^{(F)}$ is equal to the $\overline{T}^{(I)}$ in Eq. (16) if we commute the gluon fields with the quark fields [2,13]. In Eq. (37), $\overline{H}_{\mu\nu}(x,x_1,x_2,k_T,p,q,l)$ is the corresponding partonic part.

For the diagram shown in Fig. 3(a), the partonic part is given by

$$\overline{H}_{\mu\nu}^a = C\hat{H}_{\mu\nu} \frac{1}{(xp+q)^2+i\epsilon}\frac{1}{(xp+x_2p+q)^2-i\epsilon}(2\pi)\delta((xp+x_1p+k_T+q)^2)\delta(l_T^2-k_T^2)dl_T^2$$
$$= C\cdot\hat{H}_{\mu\nu}\cdot\frac{2\pi}{(2p\cdot q)^3}\cdot\delta(l_T^2-k_T^2)dl_T^2\delta\left(x+x_1-x_B-\frac{k_T^2-2q\cdot k_T}{2p\cdot q}\right)\frac{1}{x-x_B+i\epsilon}\frac{1}{x+x_2-x_B-i\epsilon}, \tag{39}$$

where C is the color factor for the diagrams in Fig. 3, and given by $C=1/6$. In Eq. (39), the δ function is from the phase space, and $\hat{H}_{\mu\nu}$ represents the spinor trace and is given by

$$\hat{H}_{\mu\nu} = (4\pi)^2\alpha_s\alpha_{em}e_q^2 \mathrm{Tr}[\gamma\cdot p\gamma_\mu\gamma\cdot(xp+x_2p+q)\gamma^\sigma\gamma\cdot l\gamma^\rho\gamma\cdot(xp+q)\gamma_\nu]p_\rho p_\sigma, \tag{40}$$

where $p_\rho p_\sigma$ is a result of our definition for the hadronic matrix element in Eq. (38).

Following the derivation of $\Delta\langle q_T^2\rangle$ in last section, first, we carry out the integrations of the parton momentum fractions by using the δ function and two poles in Eq. (39),

$$H^a \equiv \int dx dx_1 dx_2 e^{ixp^+y^-} e^{ix_1p^+(y_1^- - y_2^-)} e^{ix_2p^+y_2^-} \delta(l_T^2-k_T^2)\delta\left(x+x_1-x_B-\frac{k_T^2-2q\cdot k_T}{2p\cdot q}\right)\frac{1}{x-x_B+i\epsilon}\frac{1}{x+x_2-x_B-i\epsilon}$$
$$= (2\pi)^2\theta(y_1^- - y^-)\theta(y_2^-)e^{ix_Bp^+y^-}\delta(l_T^2-k_T^2)e^{i((k_T^2-2q\cdot k_T/2p\cdot q))p^+(y_1^- - y_2^-)}. \tag{41}$$

Similarly, we have the corresponding integrations for the interference diagram shown in Fig. 3(b)

$$H^b \equiv \int dx dx_1 dx_2 e^{ixp^+y^-} e^{ix_1p^+(y_1^- - y_2^-)} e^{ix_2p^+y_2^-} \delta(l_T^2)\delta(x+x_1-x_B)\frac{1}{x-x_B+i\epsilon}\frac{1}{x+x_1-x_B-\dfrac{k_T^2-2q\cdot k_T}{2p\cdot q}+i\epsilon}$$
$$= -(2\pi)^2\theta(y_2^- - y_1^-)\theta(y_1^- - y^-)e^{ix_Bp^+y^-}\delta(l_T^2)e^{i((k_T^2-2q\cdot k_T)/2p\cdot q)p^+(y_1^- - y_2^-)}, \tag{42}$$

and for the diagram in Fig. 3(c)

$$H^c \equiv \int dx dx_1 dx_2 e^{ixp^+y^-} e^{ix_1p^+(y_1^- - y_2^-)} e^{ix_2p^+y_2^-} \delta(l_T^2)\,\delta(x-x_B) \frac{1}{x+x_1-x_B-(k_T^2-2q\cdot k_T)/2p\cdot q - i\epsilon} \frac{1}{x+x_2-x_B-i\epsilon}$$
$$= -(2\pi)^2 \theta(y_2^-)\,\theta(y_1^- - y_2^-)\, e^{ix_Bp^+y^-}\,\delta(l_T^2)\, e^{i((k_T^2-2q\cdot k_T)/2p\cdot q)p^+(y_1^- - y_2^-)}. \tag{43}$$

Combining Eqs. (41), (42) and (43), we again have the same structure as that in Eq. (24). Therefore, like in the Drell-Yan case, we conclude that the double scattering does not result into any large nuclear size dependence in the inclusive DIS cross section. For the jet broadening defined in Eq. (34), we drop the term proportional to $\theta(y_1^- - y^-)\,\theta(y_2^-) - \theta(y_2^- - y_1^-)\,\theta(y_1^- - y^-) - \theta(y_1^- - y_2^-)\,\theta(y_2^-)$, which is localized as the single scattering. After carrying out the algebra similar to those following Eq. (26), we derive jet broadening in DIS as

$$\Delta\langle l_T^2\rangle = \left(\frac{4\pi^2\alpha_s}{3}\right) \frac{\sum_q e_q^2 T_{q/A}(x_B)}{\sum_q e_q^2 \phi_{q/A}(x_B)}, \tag{44}$$

where Σ_q sums over all quark and antiquark flavors. In Eq. (44), $\phi_{q/A}(x)$ is the normal twist-2 quark distribution inside a nucleus, and the four-parton correlation function, $T_{q/A}(x_B)$ is defined in Eq. (1a).

From Eq. (44), we conclude that based on the leading order calculation, we can uniquely predict jet broadening in DIS, without any free parameter, if the four-parton correlation function, $T_{q/A}(x_B)$, is measured in another experiment. Therefore, jet broadening in DIS can be used to test the QCD treatment of the multiple scattering in a nuclear environment. In addition, the jet broadening given in Eq. (44) is directly proportional to $T_{q/A}(x_B)$. Therefore, x_B dependence of $\Delta\langle l_T^2\rangle$ can provide immediate information on the functional form of the four-parton correlation function.

IV. DISCUSSION AND CONCLUSIONS

From Eqs. (31) and (44), both jet broadening in DIS and nuclear enhancement of average Drell-Yan transverse momentum are directly proportional to the four-parton (twist-4) correlation functions, which are defined in Eqs. (1a) and (29), respectively. Since field operators commute on the light-cone [2,13], the four-parton correlation functions, $T_{q/A}(x)$ and $T_{q/A}^{(I)}(x)$, are the same if the phase space integral are symmetric. With the model given in Eq. (2), we predict jet broadening in DIS as

$$\Delta\langle l_T^2\rangle = \left(\frac{4\pi^2\alpha_s}{3}\right)\lambda^2 A^{1/3}; \tag{45}$$

and predict nuclear enhancement of average Drell-Yan transverse momentum square as

$$\Delta\langle q_T^2\rangle = \left(\frac{4\pi^2\alpha_s}{3}\right)\lambda^2 A^{1/3}, \tag{46}$$

which is the same as that obtained in Ref. [4]. It is a direct result of the model proposed by LQS [3] that both jet broadening in DIS and nuclear enhancement of average Drell-Yan transverse momentum share the same simple expressions, as shown in Eqs. (45) and (46). In addition, Eqs. (45) and (46) tell us that at the leading order, jet broadening in DIS and nuclear enhancement of average Drell-Yan transverse momentum, $\Delta\langle q_T^2\rangle$, have the same magnitude, if the averaged initial-state gluon interactions is equal to the corresponding final-state gluon interactions, i.e., $T_{q/A}(x) = T_{q/A}^{(I)}(x)$.

From the simple expression in Eq. (45), we conclude that at the leading order, jet broadening in DIS has a strong scaling property, that it does not depend on beam energy, Q^2 and x_B. This scaling property of jet broadening in DIS is a direct consequence of LQS model for four-parton correlation functions, given in Eq. (2). However, when $x_B \leqslant 0.1$, $y^- \sim 1/(x_Bp^+)$ is no longer localized inside an individual nucleon [11,14]. Therefore, terms proportional to $\theta(y_1^- - y^-)\,\theta(y_2^-) - \theta(y_2^- - y_1^-)\,\theta(y_1^- - y^-) - \theta(y_1^- - y_2^-)\,\theta(y_2^-)$, are no longer localized and need to be kept for the jet broadening calculation if x_B is small. Consequently, the x_B scaling of jet broadening in DIS needs to be modified in small x_B region. In addition, Q^2 dependence may be modified because the four-parton correlation function $T_{q/A}(x)$ and the normal quark distribution $\phi_{q/A}(x)$ can have different scaling violation. In principle, all dependence or whole conclusion could be modified due to possible different high order corrections. Nevertheless, we believe that experimental measurements of jet broadening in DIS can provide valuable information on the strength of multiparton correlations and the dynamics of the multiple scattering. Examing the scaling property of jet broadening can provide a direct test of LQS model for four-parton correlation functions, given in Eq. (2).

Similarly, from Eq. (46), we can also conclude that nuclear enhancement of average Drell-Yan transverse momentum, $\Delta\langle q_T^2\rangle$ has a small dependence on beam energy and Q^2 of the lepton pair. Data from Fermilab E772 and CERN NA10 [7,8] demonstrate weak energy dependence. It signals that the simple model by LQS for four-parton correlation functions is reasonable and the leading order calculation given here are useful. At the same time, the observed energy dependence indicates that the high order corrections to $\Delta\langle q_T^2\rangle$ cannot be ignored [9,10].

Using Eq. (46) and data from E772 and NA10 on nuclear enhancement of average Drell-Yan transverse momentum, $\Delta\langle q_T^2\rangle$, we estimate the value of λ^2 as

$$\lambda_{\rm DY}^2 \approx 0.01\ \ \mathrm{GeV}^2, \tag{47}$$

which is at least a factor of five smaller than $\lambda^2_{\rm dijet} \approx 0.05 - 0.1$ GeV^2, estimated by LQS from momentum imbalance of the dijet data [3].

Having derived λ^2 from the dijet and the Drell-Yan data, our result in Eq. (45) provides an absolute prediction of jet broadening in DIS, without any additional free parameter. Since jet broadening in DIS and momentum imbalance of dijet are both due to the final-state multiple scattering, we will use the value of $\lambda^2_{\rm dijet}$ to predict the jet broadening as

$$\Delta\langle l_T^2\rangle \approx \left(\frac{4\pi^2\alpha_s}{3}\right)\lambda^2_{\rm dijet}A^{1/3} \approx (0.66-1.31)\,\alpha_s A^{1/3}. \quad (48)$$

On the other hand, with a smaller value of $\lambda^2_{\rm DY}$ in Eq. (47), we will predict a much smaller jet broadening

$$\Delta\langle l_T^2\rangle \approx \left(\frac{4\pi^2\alpha_s}{3}\right)\lambda^2_{\rm DY}A^{1/3} \approx 0.13\alpha_s A^{1/3}. \quad (49)$$

Direct experimental measurement on jet broadening can certainly provide an independent test of our QCD treatment of the multiple scattering. Since the dijet momentum imbalance and nuclear enhancement of average Drell-Yan transverse momentum suggest very different size of the four-parton correlation functions, it is extremely important to exam jet broadening in DIS.

Furthermore, from Eq. (44), we see that the jet broadening is directly proportional to the four-parton correlation function $T_{q/A}(x_B)$. Information on x_B dependence of the jet broadening can provide a first ever direct measurement of the functional form of four-parton correlation functions $T_{q/A}(x_B,A)$. By examing the x_B-scaling property of the jet broadening, we can test LQS model for the four-parton correlation functions. Future experiments at HERA with a heavy ion beam [11] should be able to provide much more information on dynamics of parton correlations.

In summary, we have derived analytic expressions for jet broadening in DIS and nuclear enhancement of average Drell-Yan transverse momentum in terms of universal four-parton correlation functions. Using data on dijet momentum imbalance and nuclear enhancement of average Drell-Yan transverse momentum, we predicted the size of jet broadening in DIS without any free parameter. However, because the dijet data (pure final-state multiple scattering) and the Drell-Yan data (pure initial-state multiple scattering) favor two different sizes of the four-parton correlation function, measurement of jet broadening in DIS (pure final-state multiple scattering) will provide a critical test of QCD dynamics of the multiple scattering. Since at the leading order, nuclear enhancement of average Drell-Yan transverse momentum, $\Delta\langle q_T^2\rangle_{\rm DY}$, and jet broadening in DIS, $\Delta\langle l_T^2\rangle_{\rm DIS}$ are independent of four-gluon correlation function $T_{g/A}$, measurement of $\Delta\langle q_T^2\rangle_{\rm DY}$ and $\Delta\langle l_T^2\rangle_{\rm DIS}$ provide a direct comparison between initial-state multiple scattering and final-state multiple scattering. On the other hand, jet broadening in DIS and dijet momentum imbalance are both due to pure final-state interaction. Any discrepancy with Eq. (48) signals either a contradiction between different data sets or a need for better theoretical understanding of the multiple scattering.

ACKNOWLEDGMENTS

This work was partially supported by the U.S. Department of Energy under contract No. DE-FG02-93ER40764.

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